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Electromagnetic drift waves in nonuniform quantum magnetized electron–positron–ion plasmas

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Abstract
Electromagnetic drift waves in a nonuniform quantum magnetized electron–positron–ion (EPI) plasma are studied. By using the quantum hydrodynamic equations with magnetic fields of the Wigner–Maxwell system, we obtained a new dispersion relation in which ions' motions are not considered. The positrons component (featured by the parameter ξ), density gradient of electrons, and of positrons are shown to have a significant impact on the dispersion relation. Our results should be relevant to dense astrophysical objects, e.g. white dwarf and pulsar magnetospheres, as well as low-temperature laboratory EPI plasmas.

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1. Introduction

As a new emerging area in plasma physics, quantum plasma has received a great deal of attention [1–6]. Quantum effects are well known for playing a crucial role in the behavior of the charged plasma particles when the de Broglie wavelength of the charge carriers becomes equal to or greater than the dimension of the quantum plasma system [7]. In such cases, quantum plasma behaves like a Fermi gas. They may consist of electrons, ions, positrons, holes and/or grains. Two well-known models, the Wigner model and Hartree model, are used to study quantum plasma systems. The former describes the statistical behavior of plasmas of the Wigner–Poisson system, whereas the latter describes the hydrodynamic behavior of plasmas of the Schrödinger–Poisson system [7, 8]. The quantum hydrodynamic (QHD) model describing the transport of charge, momentum and energy in plasmas has been introduced in semiconductor physics [9]. Quantum effects also appear in ultra-small electronic devices [10], astrophysics [11–13] and high intensity laser systems [14]. The

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quantum magnetohydrodynamic (QMHD) model was also obtained [15] by using the QHD model with a magnetic field based on the Wigner–Maxwell equations.

In recent years, investigations in quantum plasmas such as Landau damping [16], plasma echoes [17], surface wave [18], Debye screening [19], Bernstein–Greene–Kruskal equilibria [20], Zakharov equations [21] and stream instability [3, 22–24] have been extensively studied with quantum effects corrections. Recently, the spin properties in quantum plasmas have attracted much interest [25, 26] too. Two typical low-frequency waves, ion-acoustic waves and drift waves, are of special interest [5, 27–29]. The presence of plasma spatial gradients across a confining magnetic field leads to diamagnetic plasma currents which flow in the direction mutually perpendicular to the gradient of plasma density and the direction of the magnetic field. The diamagnetic drift currents flowing across the magnetic field produce the Lorentz $\mathbf{J} \times \mathbf{B}$ force. This force balances both the expansion force due to the plasma pressure gradient and the Coulombic force in equilibrium. The Coulombic interactions between the charged particles give rise to collective oscillations which are known as drift waves in the magnetized plasma. There are many ways to study this wave. The electromagnetic and electrostatic modes are two familiar drift modes existing in plasmas systems. Shukla and Ali have derived the dispersion relations of new electromagnetic drift modes existing in cold quantum magnetoplasmas by using quantum magnetohydrodynamic equations with and without ions' motions [29]. The usual electron–ion plasmas were used in their study. In this paper, the authors study the electromagnetic drift waves in a nonuniform quantum magnetized electron–positron–ion (EPI) plasmas. The pressure term is also to be considered in our investigation.

Using the quantum hydrodynamic equations with a magnetic field [15], and making several approximations (i.e. wavelengths are smaller than the characteristic length of the plasma inhomogeneity and the wave frequencies are much smaller than the electron Larmor frequency), we derive the dispersion relation for the low-frequency electromagnetic drift mode, taking into account the quantum corrections and the density gradient of electrons and positrons. The positrons component effects on the dispersion relation are shown to be significant by our results, which will be presented in following sections. Section 2 introduces the basic mathematical model based on the quantum hydrodynamic equations and Maxwell equations in quantum nonuniform magnetized EPI plasmas. The dispersion relation when ion motions are ignored is derived in section 3. Specific discussions on the dispersion relation are presented in section 4. And finally, we give a brief summary in section 5.

2. Mathematical model

In the present work, we consider a nonuniform quantum plasma composed of electrons, positrons and singly charged ions. The plasma is assumed to be placed in an external magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector along the z-direction. Number density $n_{j0}(x)$ is assumed to be inhomogeneous along the x-direction, where n_{j0} is the equilibrium number density of species j and $j = i, p, e$ represents ions, positrons and electrons, respectively. The quasi-neutrality condition reads as $n_{i0} + n_{p0} = n_{e0}$. The basic quantum fluid model reads

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{u}_j) = 0, \tag{1}$$

$$m_j \frac{d\mathbf{u}_j}{dt} = q_j(\mathbf{E} + \mathbf{u}_j \times \mathbf{B}) - \frac{1}{n_j}\nabla P_j + \frac{\hbar^2}{2m_j}\nabla\left(\frac{\nabla^2\sqrt{n_j}}{\sqrt{n_j}}\right), \tag{2}$$

which is supplemented by the Poisson equation

$$\nabla^2\phi = \frac{e}{\varepsilon_0}(n_e - n_i - n_p), \tag{3}$$

and Ampère's law

$$\nabla^2\mathbf{A} = \frac{e}{c^2\varepsilon_0}(n_e\mathbf{u}_e - n_p\mathbf{u}_p - n_i\mathbf{u}_i). \tag{4}$$

Here, $n_j, \mathbf{u}_j, m_j$ and P_j are the number density, velocity, mass and pressure of species j, respectively; $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the magnetic field; ϕ is the electrostatic potential, $\mathbf{A}$ is the vector potential, e is the magnitude of the electronic charge, and c is the speed of light, while ε_0 and $\hbar$ are the dielectric and scaled Planck's constants. The quantum effects are represented by the last $\hbar$-dependent term on the right-hand side of equation (2), the so-called Bohm potential. But we stress that the pressure term contains both the Fermi pressure, P_{Fj}, and the thermal pressure, P_{tj}. Since ions do not have enough time to respond to perturbations when the wave frequency is high enough, ions can be assumed to be immobile. In the next context, when we mention species j, we mean electrons and positrons only. Now we introduce the following parameters for convenience: $\chi_e = \partial \ln n_{e0}/\partial x$, $\chi_p = \partial \ln n_{p0}/\partial x$ and $\xi = n_{p0}/n_{e0}$, where χ_e (χ_p) denotes the characteristic length of the electrons (positrons) inhomogeneity and ξ denotes the positrons component. A quantity φ is assumed to have the following form:

$$\varphi = \varphi_0 + \varphi_1,$$

where φ_0 is the unperturbed value and φ_1 is a small perturbation according to $\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r} - \mathrm{i}\omega t)$. Here, ω is the wave frequency and $\mathbf{k}$ is the wave vector. Weak inhomogeneity approximation can be described by $\chi_j/k \ll 1$. ξ is assumed to be much less than 1, which means that the positrons number density is much lower than that of the electrons. An arbitrary vector $\mathbf{M}$ can been written as $\mathbf{M}_\perp + M_z\hat{\mathbf{z}}$, where $\mathbf{M}_\perp$ and M_z are the components perpendicular and parallel to the z-axis, respectively. Therefore, a set of linearized equations can be expressed as [29]

$$\frac{\partial n_{e1}}{\partial t} + \nabla_\perp\cdot(n_{e0}\mathbf{u}_{e\perp}) + n_{e0}\frac{\partial u_{ez}}{\partial z} = 0, \tag{5}$$

$$\frac{\partial n_{p1}}{\partial t} + \nabla_\perp\cdot(n_{p0}\mathbf{u}_{p\perp}) + n_{p0}\frac{\partial u_{pz}}{\partial z} = 0, \tag{6}$$

$$m_e\frac{\partial \mathbf{u}_{e\perp}}{\partial t} = e\nabla_\perp\phi - eB_0\mathbf{u}_{e\perp}\times\hat{\mathbf{z}} + \frac{\hbar^2}{4m_e}\nabla_\perp\left(\frac{\nabla^2 n_{e1}}{n_{e0}}\right) - \frac{1}{n_{e0}}\nabla_\perp P_{e1}, \tag{7}$$

$$m_p\frac{\partial \mathbf{u}_{p\perp}}{\partial t} = -e\nabla_\perp\phi + eB_0\mathbf{u}_{p\perp}\times\hat{\mathbf{z}} + \frac{\hbar^2}{4m_p}\nabla_\perp\left(\frac{\nabla^2 n_{p1}}{n_{p0}}\right) - \frac{1}{n_{p0}}\nabla_\perp P_{p1}, \tag{8}$$

$$m_e\frac{\partial u_{ez}}{\partial t} = -eE_z + \frac{\hbar^2}{4m_e}\frac{\partial}{\partial z}\left(\frac{\nabla^2 n_{e1}}{n_{e0}}\right) - \frac{1}{n_{e0}}\frac{\partial P_{e1}}{\partial z}, \tag{9}$$

$$m_p\frac{\partial u_{pz}}{\partial t} = eE_z + \frac{\hbar^2}{4m_p}\frac{\partial}{\partial z}\left(\frac{\nabla^2 n_{p1}}{n_{p0}}\right) - \frac{1}{n_{p0}}\frac{\partial P_{p1}}{\partial z}, \tag{10}$$

$$\nabla^2\phi = \frac{e}{\varepsilon_0}(n_{e1} - n_{p1}), \tag{11}$$

$$\nabla^2 A_z = \frac{e}{c^2\varepsilon_0}(n_{e0}u_{ez} - n_{p0}u_{pz}). \tag{12}$$

Here, $\nabla_\perp = \hat{\mathbf{x}}\partial/\partial x + \hat{\mathbf{y}}\partial/\partial y$, where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors along the x- and y-axes. By the way, we point out that the exact expression of the first order of the Bohm potential is

$$\frac{\hbar^2}{4m_j}\nabla\left[\frac{\nabla^2 n_{j1}}{n_{j0}} - \frac{\nabla n_{j0}\cdot\nabla n_{j1}}{n_{j0}^2} + \frac{(\nabla n_{j0})^2}{n_{j0}^3}n_{j1} - \frac{\nabla^2 n_{j0}}{n_{j0}^2}n_{j1}\right].$$

By taking account of $k_x \ll k$ and the weak inhomogeneity approximation $\chi_j \ll k$, the formula above can be simplified by keeping only the first term in the square brackets.

3. Analytical development

By carrying out the operation as done by Shukla and Ali in [29], the Poisson equation can be written as $n_{e1} = \varepsilon_0\nabla^2\phi/e + n_{p1}$. Adopting the latter in equations (7) and (8), we obtain

$$m_e\frac{\partial \mathbf{u}_{e\perp}}{\partial t} = e\nabla_\perp\phi_e - eB_0\mathbf{u}_{e\perp}\times\hat{\mathbf{z}} - \frac{1}{n_{e0}}\nabla_\perp P_{e1}, \tag{13}$$

and

$$m_p\frac{\partial \mathbf{u}_{p\perp}}{\partial t} = -e\nabla_\perp\phi_p + eB_0\mathbf{u}_{p\perp}\times\hat{\mathbf{z}} - \frac{1}{n_{p0}}\nabla_\perp P_{p1}, \tag{14}$$

with

$$\phi_e = \left(1+\lambda_{qe}^4\nabla^4\right)\phi + \frac{\hbar^2}{4m_e n_{e0} e}\nabla^2 n_{p1}, \tag{15}$$

$$\phi_p = \phi - \frac{\hbar^2}{4m_p n_{p0} e}\nabla^2 n_{p1}, \tag{16}$$

where $\lambda_{qe} = \left(\hbar^2\big/4m_e^2\omega_{pe}^2\right)^{1/4}$ is the electron quantum wavelength, $\omega_{pe} = (e^2 n_{e0}/m_e\varepsilon_0)^{1/2}$ is the electron plasma frequency. In the drift approximation $|\partial/\partial t| \ll \omega_{ce}$, we obtain the perpendicular velocities from equations (13) and (14) as

$$\begin{aligned}\mathbf{u}_{e\perp} \simeq{}& \frac{\hat{\mathbf{z}}\times\nabla_\perp\left(1+\lambda_{qe}^4\nabla^4\right)\phi}{B_0} - \frac{\hat{\mathbf{z}}\times\nabla_\perp P_{e1}}{en_{e0}B_0} + \frac{1}{B_0\omega_{ce}}\frac{\partial}{\partial t}\nabla_\perp\left(1+\lambda_{qe}^4\nabla^4\right)\phi \\ &+ \frac{\hbar^2}{4m_e eB_0}\hat{\mathbf{z}}\times\nabla_\perp\left(\frac{\nabla^2 n_{p1}}{n_{e0}}\right) + \frac{\hbar^2}{4m_e eB_0\omega_{ce}}\frac{\partial}{\partial t}\nabla_\perp\left(\frac{\nabla^2 n_{p1}}{n_{e0}}\right) \\ &- \frac{1}{eB_0 n_{e0}\omega_{ce}}\frac{\partial}{\partial t}\nabla_\perp P_{e1},\end{aligned} \tag{17}$$

$$\begin{aligned}\mathbf{u}_{p\perp} \simeq{}& \frac{\hat{\mathbf{z}}\times\nabla_\perp\phi}{B_0} + \frac{\hat{\mathbf{z}}\times\nabla_\perp P_{p1}}{en_{p0}B_0} - \frac{1}{B_0\omega_{cp}}\frac{\partial}{\partial t}\nabla_\perp\phi - \frac{1}{eB_0 n_{p0}\omega_{ce}}\frac{\partial}{\partial t}\nabla_\perp P_{p1} \\ &- \frac{\hbar^2}{4m_p eB_0}\hat{\mathbf{z}}\times\nabla_\perp\left(\frac{\nabla^2 n_{p1}}{n_{p0}}\right) + \frac{\hbar^2}{4m_p eB_0\omega_{cp}}\frac{\partial}{\partial t}\nabla_\perp\left(\frac{\nabla^2 n_{p1}}{n_{p0}}\right),\end{aligned} \tag{18}$$

where $\omega_{ce}(\omega_{cp})$ is the electron (positron) gyrofrequency. There is $\omega_{ce} = \omega_{cp} = eB_0/m_e$. Subtracting equation (6) from equation (5) and using $n_{e0}u_{ez} - n_{p0}u_{pz} = \nabla^2 A_z/\mu_0 e$, we find

$$\frac{\partial^2}{\partial t^2}\nabla^2\phi + \frac{e}{\varepsilon_0}\nabla_\perp\cdot\frac{\partial}{\partial t}(n_{e0}\mathbf{u}_{e\perp} - n_{p0}\mathbf{u}_{p\perp}) + c^2\frac{\partial}{\partial z}\nabla^2\frac{\partial A_z}{\partial t} = 0. \tag{19}$$

Using equations (9), (10), (12) and $E_z = -\partial\phi/\partial z - \partial A_z/\partial t$ from above, we get

$$\frac{c^2\nabla^2}{\omega_{pe}^2}\frac{\partial A_z}{\partial t} = \left(\frac{\partial\phi}{\partial z} + \frac{\partial A_z}{\partial t}\right)\left(1+\frac{n_{p0}}{n_{e0}}\right) + \lambda_{qe}^4\nabla^4\frac{\partial\phi}{\partial z} - \frac{1}{en_{e0}}\left(\frac{\partial P_{e1}}{\partial z} - \frac{\partial P_{p1}}{\partial z}\right). \tag{20}$$

For long wavelengths, such that $\hbar k/m \ll v_F \ll \omega/k$, the pressure term for species j can be written as $\nabla P_{j1} = m_j v_j^2 \nabla n_{j1}$, where v_j^2 denotes $v_{tj}^2 + 3v_{Fj}^2/5$: $v_{tj} = \sqrt{T_j/m_j}$ is the thermal velocity and $v_{Fj} = (\hbar/m_j)(3\pi^2 n_j)^{1/3}$ is the Fermi velocity of species j [1, 7, 6]. Using this relation, we have

$$\frac{\partial A_z}{\partial t} = -\frac{\left(1+\xi+\lambda_{qe}^4\nabla^4 - \frac{v_e^2\nabla^2}{\omega_{pe}^2}\right)\frac{\partial\phi}{\partial z} - \frac{m_e}{en_{e0}}\left(v_e^2 - v_p^2\right)\frac{\partial n_{p1}}{\partial z}}{1+\xi+\frac{c^2k^2}{\omega_{pe}^2}}. \tag{21}$$

By substituting the above formula, equations (17) and (18) into equation (19), we can write equation (19) as

$$\begin{aligned}\frac{\partial^2}{\partial t^2}\nabla^2\phi &+ \frac{e}{\varepsilon_0}\nabla_\perp\cdot\left[\frac{n_{e0}}{B_0}\hat{\mathbf{z}}\times\nabla_\perp\left(1+\lambda_{qe}^4\nabla^4\right)\frac{\partial\phi}{\partial t} + \frac{n_{e0}}{B_0\omega_{ce}}\frac{\partial^2}{\partial t^2}\nabla_\perp\left(1+\lambda_{qe}^4\nabla^4\right)\phi - \frac{n_{p0}}{B_0}\hat{\mathbf{z}}\right.\\ &\left.\times\nabla_\perp\frac{\partial\phi}{\partial t} + \frac{n_{p0}}{B_0\omega_{ce}}\frac{\partial^2}{\partial t^2}\nabla_\perp\phi - \frac{1}{eB_0\omega_{ce}}\nabla_\perp\frac{\partial^2}{\partial t^2}P_{e1} + \frac{1}{eB_0\omega_{cp}}\nabla_\perp\frac{\partial^2}{\partial t^2}P_{p1}\right] + \mathcal{F}\\ &- c^2\frac{\partial}{\partial z}\nabla^2\frac{\left(1+\xi+\lambda_{qe}^4\nabla^4 - v_e^2\nabla^2/\omega_{pe}^2\right)\partial\phi/\partial z - (m_e/en_{e0})\left(v_e^2-v_p^2\right)\partial n_{p1}/\partial z}{1+\xi+c^2k^2/\omega_{pe}^2}\\ &= 0,\end{aligned} \tag{22}$$

with

$$\begin{aligned}\mathcal{F} = \frac{e}{\varepsilon_0}\nabla_\perp\cdot\frac{\partial}{\partial t}&\left[\frac{n_{e0}\hbar^2}{4m_e eB_0}\hat{\mathbf{z}}\times\nabla_\perp\left(\frac{\nabla^2 n_{p1}}{n_{e0}}\right) + \frac{n_{e0}\hbar^2}{4m_e eB_0\omega_{ce}}\frac{\partial}{\partial t}\nabla_\perp\left(\frac{\nabla^2 n_{p1}}{n_{e0}}\right)\right.\\ &\left.+\frac{\hbar^2}{4m_p eB_0}\hat{\mathbf{z}}\times\nabla_\perp\left(\frac{\nabla^2 n_{p1}}{n_{p0}}\right) - \frac{\hbar^2}{4m_p eB_0\omega_{cp}}\frac{\partial}{\partial t}\nabla_\perp\left(\frac{\nabla^2 n_{p1}}{n_{p0}}\right)\right].\end{aligned} \tag{23}$$

Considering the drift approximation $|\partial/\partial t| \ll \omega_{ce}$ and using equation (2), we obtain $\partial n_{p1}/\partial t \simeq -\nabla_\perp\cdot(n_{p0}\hat{\mathbf{z}}\times\nabla_\perp\phi)/B_0$. In this case, $\mathcal{F}$ can be simplified as

$$\mathcal{F} \simeq -\lambda_{qe}^4 k^2 k_y^2 \xi\chi_p(\chi_e+\chi_p)\frac{\omega_{pe}^4}{\omega_{ce}^2}\phi, \tag{24}$$

which can be neglected in equation (22) in our model. The pressure terms and the last term containing n_{p1} in equation (22) can be simplified in the same way. Finally, we obtain

$$\omega^2 - \Omega_1\omega - \Omega_2 = 0, \tag{25}$$

where

$$\Omega_1 = \frac{\frac{\omega_{pe}^2}{\omega_{ce}}\chi_e k_y\left(1+k^4\lambda_{qe}^4\right) - \frac{\omega_{pe}^2}{\omega_{ce}}\left[1+\frac{k_\perp^2(v_e^2-v_p^2)}{\omega_{ce}^2}\right]\xi\chi_p k_y}{k^2+\frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1+\xi+k^4\lambda_{qe}^4+\frac{k^2v_e^2}{\omega_{pe}^2}\right)k_\perp^2 - \frac{\omega_{pe}^2}{\omega_{ce}^2}\left[\left(1-\lambda_{qe}^4k^4+\frac{4k^2v_{Fe}^2}{3\omega_{pe}^2}\right)\chi_e+\xi\chi_p\right]\mathrm{i}k_x}, \tag{26}$$

and

$$\Omega_2 = \frac{k_z^2c^2\frac{1+\xi+k^4\lambda_{qe}^4+k^2v_e^2/\omega_{pe}^2}{1+\xi+c^2k^2/\omega_{pe}^2}k^2}{k^2+\frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1+\xi+k^4\lambda_{qe}^4+\frac{k^2v_e^2}{\omega_{pe}^2}\right)k_\perp^2 - \frac{\omega_{pe}^2}{\omega_{ce}^2}\left[\left(1-\lambda_{qe}^4k^4+\frac{4k^2v_{Fe}^2}{3\omega_{pe}^2}\right)\chi_e+\xi\chi_p\right]\mathrm{i}k_x}. \tag{27}$$

The solution of equation (25) is

$$\omega = \frac{\Omega_1}{2} \pm \frac{\sqrt{\Omega_1^2+4\Omega_2}}{2}. \tag{28}$$

This is the dispersion relation of electromagnetic drift waves in a nonuniform quantum magnetized EPI plasma, where the pressure effects have been taken into account. The effects on the dispersion introduced by the positrons component are represented by the parameters χ_p and ξ, while the effects due to the pressure are represented by the parameters v_e^2 and v_p^2. With variable values of these parameters, this dispersion relation will help us comprehensively understand the electromagnetic drift waves in high-density and low-temperature quantum EPI plasmas.

A special case is as follows: $\xi = 0$, $\chi_e k_x \ll k_\perp^2$, $c^2k^2/\omega_{pe}^2 \ll 1$ and $v_e^2 = 0$. Accordingly, our results can be simplified and hence, equations (26) and (27) become

$$\Omega_1 = \frac{\frac{\omega_{pe}^2}{\omega_{ce}}\chi_e k_y\left(1 + k^4\lambda_{qe}^4\right)}{k^2 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1 + k^4\lambda_{qe}^4\right)k_\perp^2}, \tag{29}$$

and

$$\Omega_2 = \frac{k_z^2c^2\left(1 + k^4\lambda_{qe}^4\right)k_\perp^2}{k^2 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1 + k^4\lambda_{qe}^4\right)k_\perp^2}, \tag{30}$$

which are exactly identical with the results reported in [29].

4. Discussions

In the previous section, we obtained the general dispersion relation of electromagnetic drift waves. Full discussions can be given by using equation (28). In this section, some detailed discussions are presented. Equation (25) gives two electromagnetic drift modes: $\omega = \Omega_1/2 + \sqrt{\Omega_1^2 + 4\Omega_2}/2$ and $\omega = \Omega_1/2 - \sqrt{\Omega_1^2 + 4\Omega_2}/2$. The two modes become one mode when we set the parallel wave number k_z equal to zero. Then, the dispersion relation equation (28) turns to $\omega = \Omega_1 = \omega_r + \mathrm{i}\gamma$, where ω_r is the real part and $\mathrm{i}\gamma$ is the imaginary part of ω. Now, the drift mode is unstable and the growth rate is

$$\gamma = \frac{\left\{\chi_e k_y\left(1 + k^4\lambda_{qe}^4\right) - \left[1 + \frac{k_\perp^2(v_e^2 - v_p^2)}{\omega_{ce}^2}\right]\xi\chi_p k_y\right\}\frac{\omega_{pe}^4}{\omega_{ce}^3}\left[\left(1 - \lambda_{qe}^4k^4 + \frac{4k^2v_{Fe}^2}{3\omega_{pe}^2}\right)\chi_e + \xi\chi_p\right]k_x}{\left[k^2 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1 + \xi + k^4\lambda_{qe}^4 + \frac{k^2v_e^2}{\omega_{pe}^2}\right)k_\perp^2\right]^2 + \left\{\frac{\omega_{pe}^2}{\omega_{ce}^2}\left[\left(1 - \lambda_{qe}^4k^4 + \frac{4k^2v_{Fe}^2}{3\omega_{pe}^2}\right)\chi_e + \xi\chi_p\right]k_x\right\}^2}. \tag{31}$$

The value of growth rate is always positive as we have assumed that χ_e is greater than $\xi\chi_p$, implying that the wave perturbations grow as time evolves. However, equation (31) also indicates that the growth rate is approximatively proportional to χ_e^2 while ω_r is proportional to χ_e from equation (28). Note that the value of unperturbed electron number density gradient χ_e is very small, thus the growth rate is very small too. Actually, one gets

$$\frac{\gamma}{\omega_r} \simeq \frac{(\chi_e + \xi\chi_p)k_x}{k^2} \ll 1, \tag{32}$$

which implies the real frequency is much great than the growth rate.

As the diamagnetic drift $\nabla n_{e0}(x) \times \hat{\mathbf{z}}$ is along the y-direction and from equations (26) and (27), we can set wave number k_x and k_z to be equal to zero in a simple case where the parallel motion is negligible. Accordingly, one gets $\gamma = 0$ from equation (31). So the perturbation is stable. Then we have $k = k_\perp = k_y$. As a result, the dispersion relation from equation (28) becomes

$$\omega = \frac{\chi_e\left(1 + k^4\lambda_{qe}^4\right) - \left[1 + \frac{k^2(v_e^2 - v_p^2)}{\omega_{ce}^2}\right]\xi\chi_p}{k + \frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1 + \xi + k^4\lambda_{qe}^4 + \frac{k^2v_e^2}{\omega_{pe}^2}\right)k}\frac{\omega_{pe}^2}{\omega_{ce}}. \tag{33}$$

In order to see how quantum effects corrections and positrons component affect the dispersion relation of electromagnetic drift wave more explicitly, we plot the relationship between the

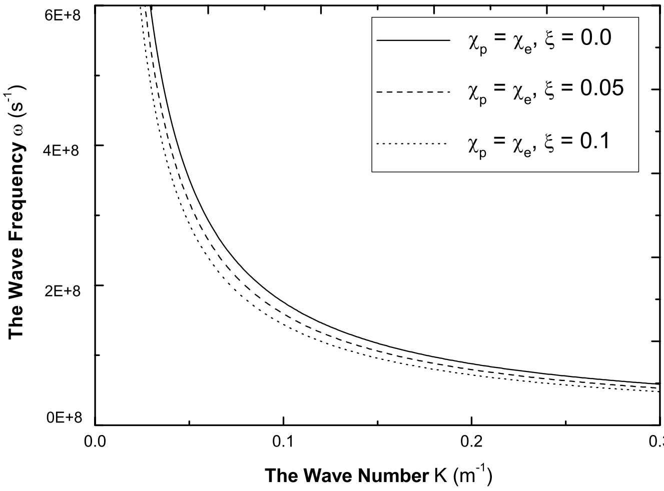

Figure 1. The wave frequency of electromagnetic drift wave versus the wave number. To illustrate the effects due to positron component, we have used the following constants and parameters: $e = 1.9 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg, $\hbar = 1.055 \times 10^{-34}$ J s, and $\varepsilon_0 = 8.854 \times 10^{-12}$ F m^{-1}; number density $n_{e0} = 5.0 \times 10^{30}$ m^{-3} and magnetic field $B_0 = 10^2$ T; number density gradients $\chi_e = \chi_p = 10^{-6}$ m^{-1}.

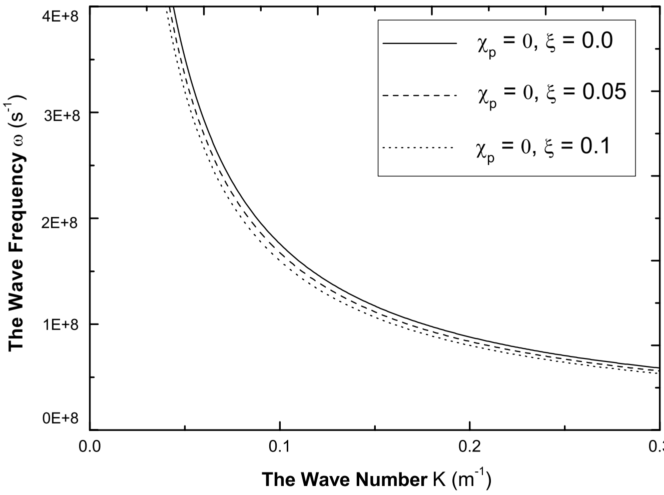

Figure 2. The wave frequency of electromagnetic drift wave versus the wave number. Number density gradients $\chi_e = 10^{-6}$ m^{-1} and $\chi_p = 0$. All other constants and parameters are the same in figure 1.

frequency ω and the wave number k in figures 1–3, where we have omitted the thermal velocities. Although we have mentioned that the pressure terms contained both the Fermi pressure and the thermal pressure, we now stress that our model should be relevant when the following ordering on the temperatures is satisfied: $T_e \ll T_{Fe}, T_p \leqslant T_e$. This condition can be easily satisfied in some dense astrophysical environments such as white dwarfs and neutron stars. In these dense astrophysical circumstances, the electron number density and the

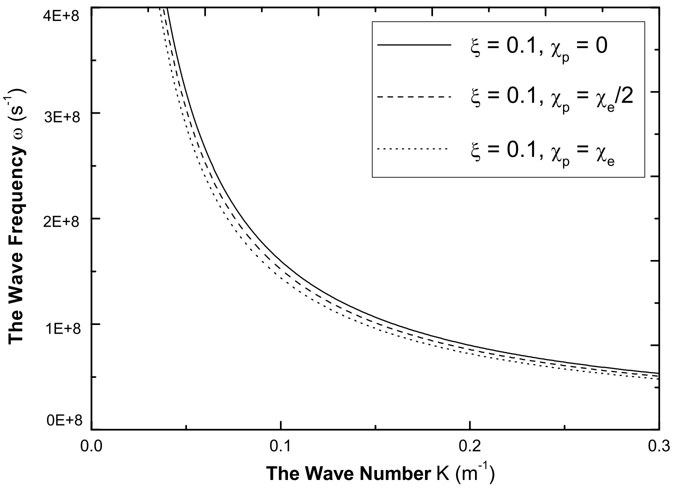

Figure 3. The wave frequency of electromagnetic drift wave versus the wave number when $\xi = 0.1$. Number density gradients $\chi_e = 10^{-6}\ \mathrm{m}^{-1}$ and $\chi_p = 0$ (solid curve), $\chi_p = \chi_e/2$ (dashed curve) and $\chi_p = \chi_e$ (dotted curve). All other constants and parameters are the same in figure 1.

magnetic field are huge enough so that quantum effects will become remarkable and dominant [2, 7]. The thermal effects can be safely neglected.

The effects of the positrons component on the dispersion relation are shown in figure 1. The figure shows that the frequency decreases with increasing value of ξ. In fact, it is not hard to find from equation (22) that

$$\partial\omega/\partial\xi < 0. \tag{34}$$

Therefore, the wave frequency is a monotonic function that decreases as parameter ξ increases, as confirmed by figure 1.

Figure 2 shows the dispersion relation when we set the positrons number density gradient χ_p equal to zero. Our purpose is to illustrate how the electrons number gradient and positrons component affect the dispersion relation when the positron density gradient effects are excluded. In this case, we also have $\partial\omega/\partial\xi < 0$, and figure 2 confirms that. Furthermore, we obtain that there is

$$\frac{\partial\omega}{\partial\chi_p} = -\frac{\left[1 + \frac{k^2(v_e^2 - v_p^2)}{\omega_{ce}^2}\right]\xi}{k + \frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1 + \xi + k^4\lambda_{qe}^4 + \frac{k^2 v_e^2}{\omega_{pe}^2}\right)k}\frac{\omega_{pe}^2}{\omega_{ce}} < 0 \tag{35}$$

from equation (33), suggesting that the wave frequency decreases monotonically as χ_p increases. This is confirmed by figure 3.

5. Summary

In conclusion, we have investigated the electromagnetic drift waves in nonuniform quantum magnetized EPI plasmas on the basis of the QHD model with magnetic field. The genetic dispersion relation was obtained. In our calculations, we have taken account of the thermal and Fermi pressure terms. Besides, a parameter $\xi = n_{p0}/n_{e0}$ is introduced to represent the positrons component. Our results indicate that the drift wave is unstable when the wave vector $\mathbf{k}$ has a component along the x-axis and the growth rate is approximately proportional to the square of the electrons number density gradient. On the contrary, the wave is stable when the wave vector **k** has only a component along the y-direction and then the wave frequency monotonically increases as ξ decreases. The density gradient of positrons χ_p also affects the dispersion and the wave frequency is proved to be a monotonic function that decreases as χ_p increases. The effect of the large parallel component of the wave vector and the spin properties will be the focus of our future research. Finally, we point out that the results of the present paper can be of particular significance for dense astrophysical environments, as well as some low-temperature and high-density plasmas systems.

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